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# Thermal pulse propagation and dispersion in laminar flow within conduits of finite wall conductivity

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**Abstract**—The temporal and spatial characteristics of thermal pulses travelling in conduits of finite wall conductivity are studied using a multiple time scale expansion. Values for the effective dispersion coefficient of these pulses for parabolic laminar flows within both tubes and parallel plate channels are derived and some experimental observations are made to confirm the prediction that the pulses propagate at a sub-mean flow velocity. Several analogues are drawn between the presently reported results and know phenomena in chromatography. Copyright © 1996 Elsevier Science Ltd.

## 1. INTRODUCTION

Heat transfer in thermally conducting fluids moving past a thermally conducting solid phase has received considerable attention in the literature. In particular, the work of Carbonell and colleagues [1–3] in the 1980s greatly increased the understanding of the dispersion–convection characteristics of superimposed temperature variations in flows through packed beds. These studies, which used a spatial averaging technique to obtain approximate solutions of the governing energy (or diffusion) equations both in the fluid and the bounding solid, clearly showed that the temperature changes generally propagate at equal velocities in the fluid and solid phases within a packed bed and that this propagation velocity is less than the mean fluid speed. Later work by Yuan *et al.* [4] further supported these earlier results and showed that the analysis of the problem is greatly simplified by introducing a moving coordinate system translating at the propagation speed of the thermal front through the packed bed. All of these studies used a transverse averaging procedure to obtain solutions of the governing equations and based their packed bed studies on a capillary model with finite wall thickness and thermal conductivity. Such problems are very reminiscent of the related problem of gas chromatography in coated capillaries as first developed by Golay [5], where one also finds that, due to wall interactions, adsorbable components added to a carrier gas propagate more slowly down the tubes than the mean carrier speed [6]. These studies on thermal dispersion are also very closely related to dispersion–convection problems through porous media with surface chemical reactions as studied by Shapiro and Brenner [7] also using a spatial averaging approach.

About a year ago we reported [8] on a related study which dealt with thermal pulse propagation in laminar flow within small diameter tubes for both insulating walls and walls of infinite thermal conductivity. The insulating wall case was shown to be essentially identical with the classical Taylor problem for solute dispersion in capillaries [9], except that the thermal diffusion coefficient appearing in the Peclet number is replaced by the solute diffusion coefficient. The thermal pulses were observed to move down the tube at the mean flow speed when the wall is a good insulator and to disperse rapidly at higher Peclet numbers. The analysis employed there was based on a multiple time scale expansion using the Graetz number as a small parameter. Earlier use of this technique to related problems in dispersion has been made by Pismen [10], Kurzweg [11] and Mauri [12].

It is our purpose here to examine, in some detail, the problem of thermal pulse propagation in laminar flow within conduits of both circular and rectangular cross-section. The conduit walls are assumed to have finite thermal conductivity and are insulated from the outside. The model used will be like that employed in references [2–4]. However, unlike these earlier inves-

**NOMENCLATURE**

$a$ tube inner radius or channel half-width
$A, B, C, D, G, H, Q$ functions involving $\varepsilon, \eta, \sigma, \mu$ and $f$
$b$ tube outer radius
$c$ initial temperature pulse half-length
$c_f, c_w$ specific heat of fluid and wall
$f$ thermal pulse propagation speed in units of $u_m$
$F$ temperature distribution for an initial rectangular pulse
$F_{max}$ maximum pulse temperature
$g$ axial velocity profile
$k_f, k_w$ thermal conductivity of fluid and wall
$L$ axial scale length (typically conduit length)
$L_1, L_2$ thermocouple locations
$Pe$ Peclet number $(au_m/\kappa_f)$
$r$ transverse coordinate
$t$ time
$T_f, T_w$ temperature of fluid and conduit wall
$T_{fn}, T_{wn}$ temperature terms in series expansions
$u_m$ mean axial fluid velocity
$z$ axial coordinate
$Z$ non-dimensional coordinate used in multiscale expansion.

Greek symbols
$\gamma$ Graetz number $(aPe/L)$
$\varepsilon$ wall thickness parameter $(b/a)$
$\eta$ non-dimensional transverse coordinate
$\kappa_f, \kappa_w$ thermal diffusivity of fluid and wall
$\kappa_{eff}$ effective thermal dispersion (or diffusion) coefficient
$\mu$ ratio of fluid to wall thermal conductivity
$\xi$ hybrid non-dimensional axial coordinate
$\rho_f, \rho_w$ density of fluid and wall
$\sigma$ ratio of fluid–wall thermal diffusivity
$\tau$ non-dimensional time $(\kappa_f t/a^2)$
$\tau_{max}$ non-dimensional time at peak of thermal pulse
$\psi$ time coordinate used in multiscale expansion.

Subscripts
f fluid
w conduit wall.

tigations, we will not be concerned with trying to simulate what happens in packed beds, but rather will concentrate on pinpointing the propagation and dispersion characteristics of rectangular thermal pulses propagating along conduits. We will solve the problem using the multiple time scale approach, which is not only more rigorous than a spatial averaging technique, but also requires considerably less computational effort to arrive at an effective dispersion coefficient for such pulses. The multiple time scale approach does require that the Graetz number be small compared to unity. This is typically the case under most experimental conditions including the ones discussed in refs. [3, 4] and the ones to be reported on below.

Of specific interest to us will be a determination of the effective axial thermal dispersion coefficient for heat pulses convected along conduits of different cross-section. After developing the appropriate effective dispersion coefficients, we will study the time history of injected rectangular heat pulses and determine the propagation speed of these pulses relative to the mean fluid speed. The relationship between the characteristics of such pulses and the known behaviour of concentration pulses in the related field of gas chromatography will also be discussed. Finally, experimental measurements of the predicted sub-mean speed propagation of thermal pulses will be verified experimentally using water flowing in a small diameter copper tube of finite wall thickness, insulated on the outside. The results will confirm that thermal pulses travel slower than the mean fluid speed for finite thickness walls and that this speed can be used to predict the ratio of heat capacity of fluid to wall material. Unlike those in gas or liquid chromatography, the observed thermal pulses will generally broaden and decrease in amplitude rapidly in their passage along the conduit.

## 2. FORMULATION OF THE PROBLEM

Consider a conduit of either circular or parallel plate cross-section and let a steady laminar flow exist within it. Such flows have their axial velocity dependent only on the cross steam direction so that $u = u_m g(r)$, with $u_m$ being the mean flow speed and $r$ the cross-stream coordinate. The distribution $g(r)$ has known parabolic shapes. The bounding walls of the conduit have finite thermal conductivity and finite thickness. At $t = 0$ a temperature pulse is superimposed on the otherwise constant temperature flow. The equations governing the temporal and spatial behaviour of this pulse in a coordinate system moving down the conduit at fraction $f$ of the mean flow velocity are:

$$\frac{\partial T_f}{\partial \tau}+Pe[g(\eta)-f]\frac{\partial T_f}{\partial \xi}=\nabla^2 T_f+\frac{\partial^2 T_f}{\partial \xi^2} \quad (1)$$

within the fluid and

$$\sigma\frac{\partial T_w}{\partial \tau} Pe f\sigma\frac{\partial T_w}{\partial \xi}=\nabla^2 T_w+\frac{\partial^2 T_w}{\partial \xi^2} \quad (2)$$

within the bounding wall. Here $\sigma=\kappa_f/\kappa_w=(k_f\rho_w c_w)/(k_w\rho_f c_f)$ is the thermal diffusivity ratio of fluid to wall material and $\nabla^2$ that part of the Laplacian involving the non-dimensional transverse coordinate $\eta=r/a$. In this formulation we have employed the non-dimensional time

$$\tau=t\frac{\kappa_f}{a^2}=\psi/\gamma^2 \quad (3)$$

and a hybrid non-dimensional axial coordinate

$$\xi=(z/a)-Pef\tau=Z/\gamma. \quad (4)$$

The quantity $Pe=u_m a/\kappa_f$ is the Peclet number and $\gamma=(a/L)Pe$ the Graetz number, with $z$ being the axial coordinate and $L$ a representative axial length. The Graetz number is generally a small quantity under most experimental conditions and forms the basis for our series expansion to be presented.

The corresponding set of boundary conditions are the derivative requirements

$$\frac{\partial T_f}{\partial \eta}=0 \text{ at } \eta=0 \quad \frac{\partial T_w}{\partial \eta}=0 \text{ at } \eta=\varepsilon\left(=\frac{b}{a}\right) \quad (5)$$

and the interfacial conditions of temperature and flux continuity

$$T_f=T_w \quad \mu\frac{\partial T_f}{\partial \eta}=\frac{\partial T_w}{\partial \eta} \quad (6)$$

at the fluid solid interface at $\eta=1$. Here $\mu=k_f/k_w$ is the fluid–wall conductivity ratio.

To solve the above set of equations analytically by the multiple time scale expansion procedure we use a series expansion in powers of Graetz number. Such a solution should be rapidly convergent whenever $\gamma$ is a small parameter. Applying this method, we first introduce the temperature expansions

$$T_f=T_{f0}+\gamma T_{f1}+\gamma^2 T_{f2}+ \quad \text{for} \quad 0<\eta<1 \quad (7)$$

and

$$T_w=T_{w0}+\gamma T_{w1}+\gamma^2 T_{w2}+ \quad \text{for} \quad 1<\eta<\varepsilon(=b/a). \quad (8)$$

Next, substituting these expansions into equations (1) and (2) and collecting terms in like powers of $\gamma$, leads to a hierarchy of linear differential equations, each of which can be solved in closed form. Applying the boundary and interfacial conditions to each of these solutions, leads to an accurate description of the temporal and spatial behaviour of thermal pulses within such conduit flows. We here carry out two explicit calculations for the already mentioned geometries. As will be seen below, the results of these calculations show that such thermal pulses generally travel at a fraction of the mean flow velocity and that the axial thermal dispersion coefficient and hence the pulse broadening, increases dramatically with increasing Peclet number.

## 3. TEMPERATURE PULSE WITHIN A TUBE OF CIRCULAR CROSS-SECTION

We begin our evaluation of the equation hierarchy resulting from substituting equations (7) and (8) into equations (1) and (2), for the case of Poiseuille flow in a pipe of inner radius $r=a$ and outer radius $r=b$. The appropriate steady velocity profile here equals $g(\eta)=2(1-\eta^2)$. The zeroth order term in the above perturbation series expansion yields the very simple solution

$$T_{f0}=T_{w0}=F(Z,\psi) \quad (9)$$

where $F$ is a function independent of the non-dimensional radial coordinate $\eta$. This result does not depend on the shape of the axial velocity profile being used. Next, the first order corrections can be found by solving

$$\nabla^2 T_{f1}=Pe(2-f-2\eta^2)\frac{\partial T_{f0}}{\partial Z} \quad (10)$$

and

$$\nabla^2 T_{w1}=-\sigma f Pe\frac{\partial T_{w0}}{\partial Z}. \quad (11)$$

These have the analytic solutions

$$T_{f1}=\frac{Pe}{4}\frac{\partial F}{\partial Z}[(2-f)\eta^2-\eta^4/2] \quad (12)$$

and

$$T_{w1}=\frac{Pe}{2}\frac{\partial F}{\partial Z}[\sigma f(-\eta^2/2+\varepsilon^2\ln\eta)+3/4+f(\sigma-1)/2]. \quad (13)$$

For these solutions to satisfy the flux continuity condition at the fluid–wall interface it follows that the coordinate system must translate at the unique axial speed of

$$f=\frac{1}{1+\frac{\sigma}{\mu}(\varepsilon^2-1)} \quad (14)$$

where $\sigma/\mu=(\rho_w c_w)/(\rho_f c_f)$ is the wall–fluid heat capacity ratio and again $\varepsilon=b/a$, the wall thickness. For an insulating wall one thus requires a coordinate system moving with the mean flow velocity, as in the Taylor case [8, 9], but for conducting walls $f$ will generally be less than one.

Finally we solve the set of equations in the hierarchy for the derivatives of $T_{f2}$ and $T_{w2}$. We find after a small amount of manipulation that

$$\frac{\partial T_{f2}}{\partial \eta} = \frac{Pe^2}{16}\frac{\partial^2 F}{\partial Z^2}[(2-f)^2\eta^3 - 5(2-f)\eta^5/3 + \eta^7/2] + \frac{\eta}{2}\left[\frac{\partial F}{\partial \psi} - \frac{\partial^2 F}{\partial Z^2}\right] \quad (15)$$

and

$$\frac{\partial T_{w2}}{\partial \eta} = -\frac{\sigma f Pe^2}{2}\frac{\partial^2 F}{\partial Z^2} \langle \sigma f[\varepsilon^2\eta(\ln\eta - 1/2)/2 + \eta/4 - \eta^3/8] + (3/4 - f/2)\eta/2\rangle + \frac{\eta}{2}\left[\sigma\frac{\partial F}{\partial \psi} - \frac{\partial^2 F}{\partial Z^2}\right] + \frac{Q}{\eta} \quad (16)$$

where

$$Q = \frac{\varepsilon}{4}(\sigma f Pe^2)\frac{\partial^2 F}{\partial Z^2}\left[\sigma f\left(\varepsilon^3\ln\varepsilon + \frac{\varepsilon}{2} - \frac{3\varepsilon^3}{4}\right) + \varepsilon\left(\frac{3}{4} - \frac{f}{2}\right)\right] - \frac{\varepsilon^2}{2}\left(\sigma\frac{\partial F}{\partial \psi} - \frac{\partial^2 F}{\partial Z^2}\right). \quad (17)$$

Upon applying the flux continuity condition at $\eta = 1$ this leads to the partial differential equation for $F(Z, \psi)$ of

$$\frac{\partial F}{\partial \psi} = \kappa_{eff}\frac{\partial^2 F}{\partial Z^2} \quad (18)$$

with $\kappa_{eff}$ being the effective non-dimensional axial thermal dispersion coefficient, expressed in units of $\kappa_f$, and given by

$$\kappa_{eff} = f[1 + A - Pe^2(B + C + D)] \quad (19)$$

with

$$A = (\varepsilon^2 - 1)/\mu \quad (20)$$

$$B = (7 - 14f + 6f^2)/48 \quad (21)$$

$$C = \sigma f(1-f)(1 - 4\varepsilon^2 + 3\varepsilon^4 - 4\varepsilon^4\ln\varepsilon)/[8(\varepsilon^2 - 1)] \quad (22)$$

and

$$D = (-2f^2 + 5f - 3)/8. \quad (23)$$

From this result one can conclude that the temperature variation in a travelling thermal pulse is governed essentially by a one-dimensional diffusion equation having an effective thermal dispersion coefficient which is typically much larger than $\kappa_f$, especially when the Peclet number has an appreciable value. This result can be shown to be equivalent to the Zanotti–Carbonell result given in equation (53) of ref. [3], after considerable manipulations and after appropriate conversions are made reconverting from a porous bed to pipe geometry. Note that for walls of zero thickness ($\varepsilon = 1$) one finds that $\kappa_{eff}$ reduces to the Taylor result [9]

$$\kappa_{eff} = 1 + (Pe^2/48). \quad (24)$$

Typically this limiting value is lower than that existing for walls of finite thermal conductivity and thickness, so that in the latter case the thermal pulses will generally decay in amplitude and widen more rapidly in their passage along the tube. In Fig. 1 we give a plot of $\kappa_{eff}$ vs Peclet number for several different wall thicknesses. A fluid–wall combination corresponding to $\mu = \sigma = 1$ was chosen for this plot. Note the very rapid increase in the dispersion coefficient with increasing *Pe*. The effective thermal diffusivity is also observed to increase with an increase in wall thickness. This trend however reverses itself as the walls become very thick.

To see what happens to an actual thermal pulse as a function of spatial position and time, we next solve equation (18) for an initial rectangular pulse input of the form

$$F(Z,0) = \begin{cases} 1 & \text{for } -\gamma c/a < Z < \gamma c/a \\ 0 & \text{for all other } Z. \end{cases} \quad (25)$$

That is, one introduces a heated constant temperature slug of length $2c$ centered on the origin at $z = 0$ at zero time. An analytic solution of the equation for this case is

$$F = \frac{1}{2}\left[\operatorname{erf}\left(\frac{c - z + a\,Pe\tau f}{2a\sqrt{\kappa_{eff}\tau}}\right) + \operatorname{erf}\left(\frac{c + z - a\,Pe\tau f}{2a\sqrt{\kappa_{eff}\tau}}\right)\right]. \quad (26)$$

In Fig. 2 we have plotted this function for three different values of non-dimensional time $\tau = t\kappa_f/a^2$ for water flowing in a copper tube with $\varepsilon = b/a = 1.2$, $c/a = 50$ and $Pe = 50$. For this fluid–wall combination the handbook values for diffusivity and conductivity ratios are $\sigma = 0.001226$ and $\mu = 0.001496$. One sees essentially a dispersing Gaussian shaped temperature pulse with a drop in pulse maximum and a widening of the profile noted, as the residence time increases. The rate at which the pulse broadens generally increases with increasing Peclet number since $\kappa_{eff}$ becomes large. However, as the value of *Pe* gets low enough, this trend is reversed, since then the fluid residence time in the tube becomes large. Such a dual effect can be demonstrated more clearly by looking at the maximum value of $F$ at the peak of these pulses. The peaks occur at a fixed point $L$ along the tube when

$$\tau_{max} = L/a\,Pe f \quad (27)$$

and has the value

$$F_{max} = \operatorname{erf}\left[\frac{c\sqrt{f Pe}}{2\sqrt{a\kappa_{eff}L}}\right]. \quad (28)$$

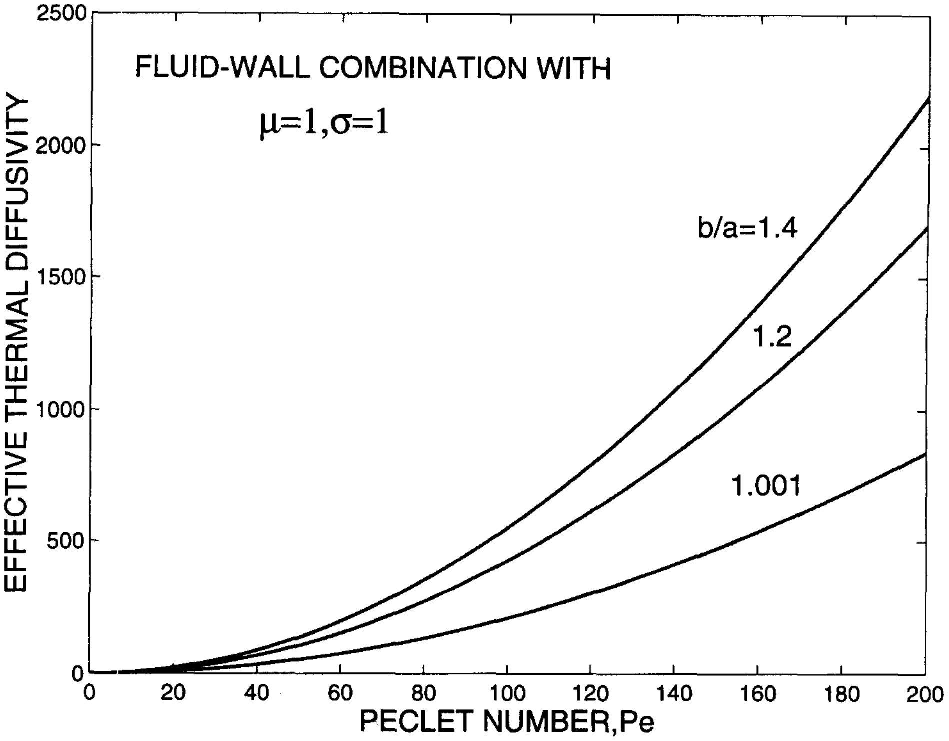


Fig. 1. Effective thermal dispersion coefficient vs Peclet number for different tube wall thickness.

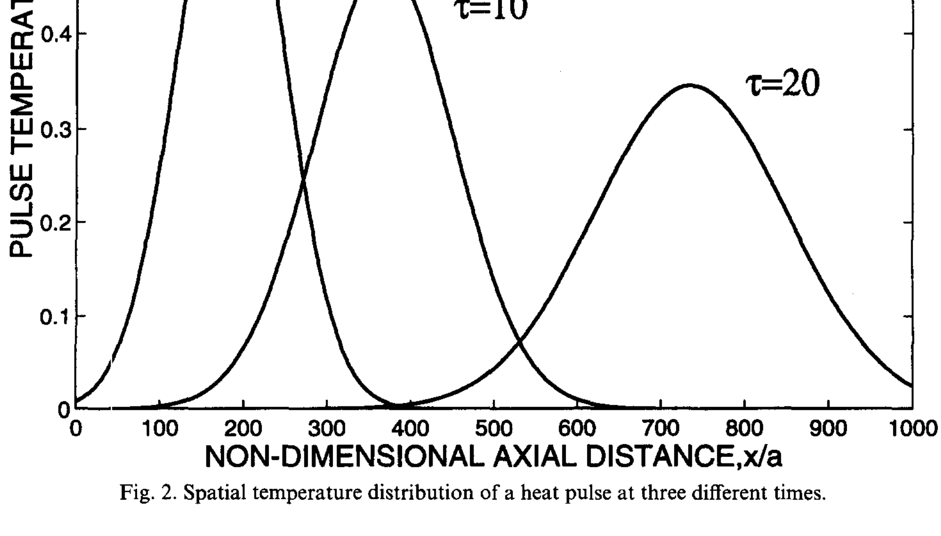


Fig. 2. Spatial temperature distribution of a heat pulse at three different times.

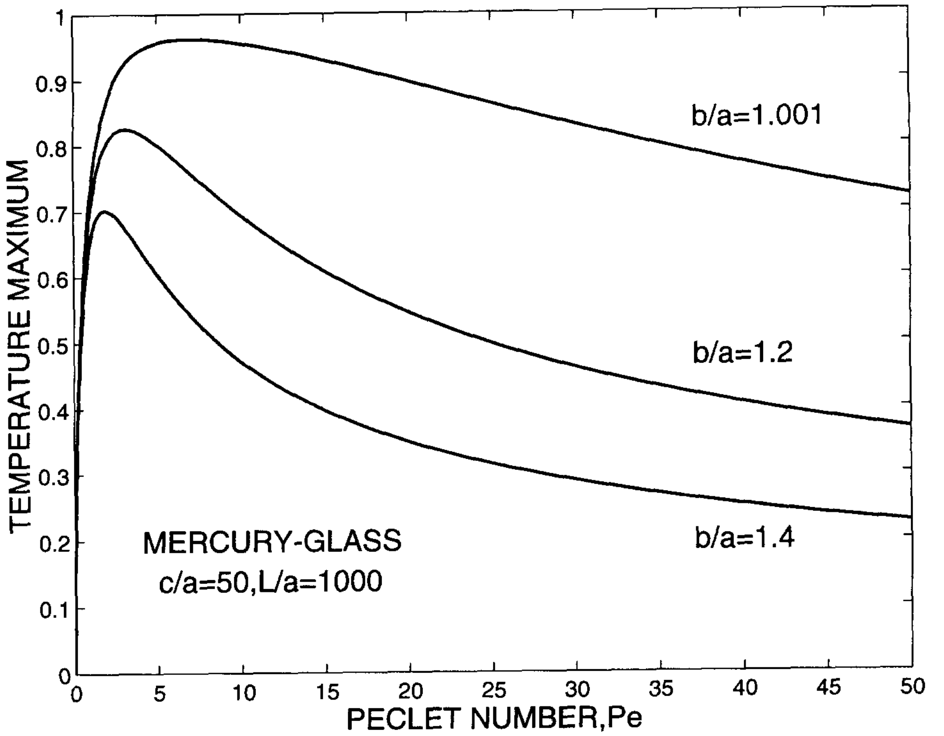


Fig. 3. Value of temperature maximum as a function of Peclet number.

This maximum becomes smaller with increasing $\kappa_{\text{eff}}$ and larger with increasing $Pe$. A plot of equation (28) is shown in Fig. 3 for a mercury–glass combination where $\sigma = 12.77$ and $\mu = 10.09$. The results clearly show that the pulse height (and hence the reciprocal of the pulse width) has a maximum, which depends on the wall thickness parameter $\varepsilon$, at an intermediate value of $Pe$. This result is consistent with the Van Deemter formula in chromatography and the Golay extension which predicts that the smallest pulse broadening (and hence best contaminant pulse resolution) occurs at intermediate fluid carrier velocities [5, 6].

## 4. THERMAL PULSE IN A FLAT PLATE CHANNEL

To see what influence a change in geometry has on the above observations, we next look at thermal pulse propagation and dispersion in a flat plate channel array. The distance between the parallel plates is taken as $2a$ and the plate walls have thickness $2(b-a)$. The axial velocity in the central channel is $g(\eta) = (3/2)(1-\eta^2)$. Applying the same multiple time scale procedure, one this time finds the solutions

$$T_{f0} = T_{w0} = F(Z, \psi) \tag{29}$$

$$T_{f1} = Pe\frac{\partial F}{\partial Z}[(3/2-f)\eta^2/2-\eta^4/8] \tag{30}$$

$$T_{w1} = Pe\frac{\partial F}{\partial Z}[\sigma f(\varepsilon\eta-\eta^2/2+(5/8-f/2-\varepsilon\sigma f+\sigma f/2) \tag{31}$$

$$\frac{\partial T_{f2}}{\partial \eta} = Pe^2\frac{\partial^2 F}{\partial Z^2}[(3/2-f)^2\eta^3/6-7(3/2-f)\eta^5/40 + 3\eta^7/112]+\eta\left[\frac{\partial F}{\partial \psi}-\frac{\partial^2 F}{\partial Z^2}\right] \tag{32}$$

$$\frac{\partial T_{w2}}{\partial \eta} = \frac{\partial^2 F}{\partial Z^2}[G(\sigma f Pe)^2+(\varepsilon-\eta)(\sigma f Pe^2 H+1)] - \frac{\partial F}{\partial \psi}[\sigma(\varepsilon-\eta)] \tag{33}$$

with

$$G = \varepsilon^3/3-\eta^2(\varepsilon/2-\eta/6) \tag{34}$$

and

$$H = 5/8-f(1/2+\varepsilon\sigma-\sigma/2). \tag{35}$$

The temperature and flux continuity conditions on $T_{f1}$ and $T_{w1}$ at $\eta = 1$ dictates that

$$f = \frac{1}{\left[1+\frac{\sigma}{\mu}(\varepsilon-1)\right]}. \tag{36}$$

Note that this coordinate translation speed differs slightly in form from that given by equation (14) for a tube. An examination of these two different values of $f$ indicates that $f$ can be thought of as a number representing that fraction of the heat of a thermal pulse contained in the fluid compared to that contained in the wall–fluid combination. For the flat plate channel case we see that this ratio equals

$$\text{ratio} = \frac{a\rho_f c_f}{[(b-a)\rho_w c_w + a\rho_f c_f]} \quad (37)$$

which agrees with the value of $f$ given by equation (36). For the case of a circular pipe conduit this ratio becomes

$$\text{ratio} = \frac{\pi a^2 \rho_f c_f}{[\pi(b^2-a^2)\rho_w c_w + \pi a^2 \rho_f c_f]} \quad (38)$$

which matches exactly the value of $f$ given by equation (14). Drawing the analogy between chromatography and the present heat dispersion study, one also sees that $f$ plays the same role as the retention factor there which measures the ratio of the molecules in the mobile phase compared to the total number of molecules.

Using equations (32) and (33) and imposing flux continuity at $\eta = 1$, yields a one dimensional diffusion equation of the form given by equation (18), except that the effective dispersion coefficient now assumes the value

$$\kappa_{\text{eff}} = f\left\langle 1 + \frac{(\varepsilon-1)}{\mu} + Pe^2\left[\left(\frac{-f^2}{6} + \frac{13f}{40} - \frac{39}{280}\right) + \frac{\sigma f^2}{\mu}((\sigma\varepsilon + 1/2)(1-\varepsilon) + \sigma(\varepsilon^3-1)/3) + 5f\sigma(\varepsilon-1)/8\mu\right]\right\rangle. \quad (39)$$

An examination of this result shows that the dispersion coefficient again increases rapidly with Peclet number and wall thickness. In the limit of zero wall thickness ($\varepsilon = 1$) one finds

$$\kappa_{\text{eff}} = 1 + \frac{2\,Pe^2}{105} \quad (40)$$

which is seen to be quite close to the Taylor result for round tubes, where the coefficient is 1/48 and relates to one of the asymptotic values found by Watson [13], in his study of contaminant dispersion in flat plate channels for oscillatory flow. The $Pe$ and $\varepsilon$ dependence of $\kappa_{\text{eff}}$ predicted by equations (39) and (19) is generally quite similar. This is supported by Fig. 4 where we have plotted $\kappa_{\text{eff}}$ for both geometries considered using the water–copper combination at $\varepsilon = 1.2$. The two curves are very close to each other over the entire range of $Pe$ considered and thus lend support to the earlier assumptions made by Carbonell [1–3] and other that capillary models can be used to approximate the much more complicated flows occurring in packed beds.

Since thermal pulses in these parallel plate channels behave nearly identically to those already presented for tubes, it will not be necessary to present such extra graphs here.

## 5. EXPERIMENTAL RESULTS

To support the above findings of pulse propagation and dispersion in conduits with finite wall conductivity, we undertook a series of experiments using a 2.5 m long copper tube of inner radius $a = 0.167$ cm and $\varepsilon = b/a = 1.43$. The tube was insulated along its outer surface with styrofoam and the working fluid was water. A piston pump, used for chromatographic investigations, was attached at the left end and an injection port for adding hot water was also installed near this end. A schematic of our experimental setup can be seen in Fig. 5. For a typical run the water in the pipe (whose right end was open to a drain) was first equilibrated to room temperature for several hours while the displacement pump was not running. Then a slug of hot water was injected by means of a syringe at the injection port. Typically about 1.5 $\text{cm}^3$ of water were added. Next the pump was turned on to produce a constant velocity axial flow of the Poiseuille type and the temperature monitored via thermocouples at two points $L_1 = 50$ cm and $L_2 = 200$ cm down the tube as a function of time. Results of a typical run are shown in Fig. 6. For this case the flow rate produced by the pump was $Q_o = 1.91\ \text{cm}^3\ \text{min}^{-1}$. The temperature pattern observed is just as predicted, with the pulse widening and decreasing in amplitude in its passage down the tube. The Peclet number for this run was 43.3 which yields a value of $\gamma = 0.145$ for the Graetz number at station one and corresponds to a mean flow velocity of $u_m = 0.363\ \text{cm s}^{-1}$. The time elapsed for the pulse maximum to travel from $L_1$ to $L_2$ was $\Delta t = 754.5$ s. From this measurement the pulse propagation speed was determined to be

$$f = \pi(L_2 - L_1)a^2/(Q_o\Delta t) = 0.547 \quad (41)$$

and from this follows in turn, via equation (14), that $\sigma/\mu = 0.792$ for this experiment. This result is quite close to the value of 0.822 obtained from the handbooks for the heat capacity ratio between copper and water and suggests that measurements, such as the present, can be used to obtain quantitative values for material heat capacities. Also, working backwards using equation (28), one finds for the same value of $f$ that the initial injected slug temperature at the beginning of the run must have been 10.3°C above ambient for a slug length estimated to have been $2c = 17$ cm. Since the injected heated fluid slug was considerably warmer than this value when first injected one must conclude that the initial cooling before the flow is started after injection must be quite rapid. There appears to be relatively little heat loss once the pulse has started its journey down the insulated conduit.

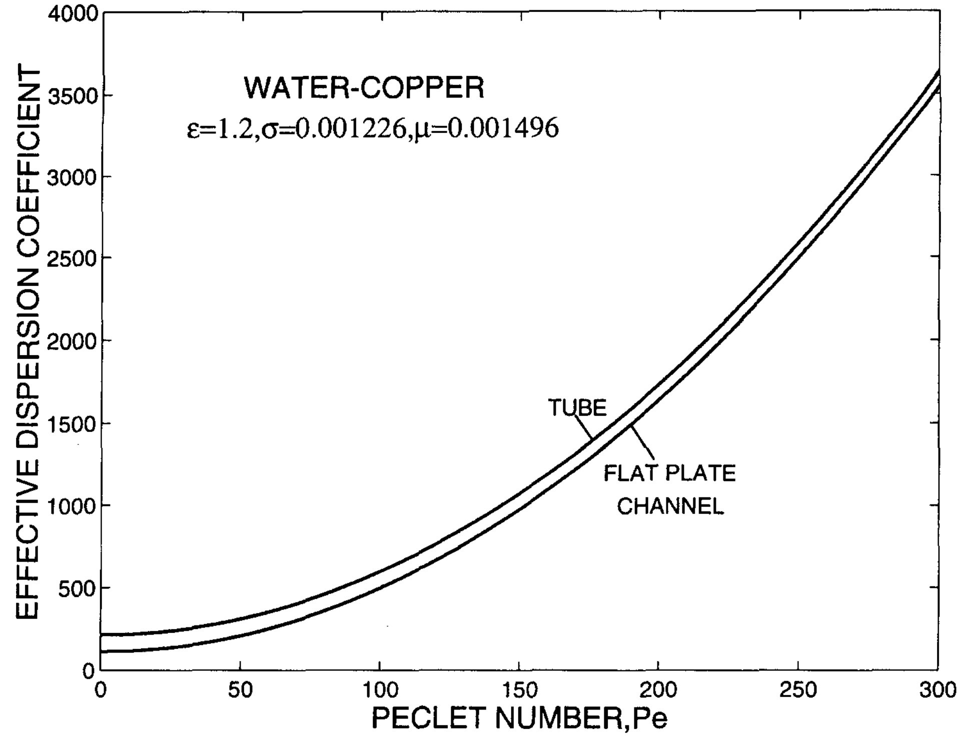


Fig. 4. Comparison of effective thermal dispersion coefficient for different cross-section conduits.

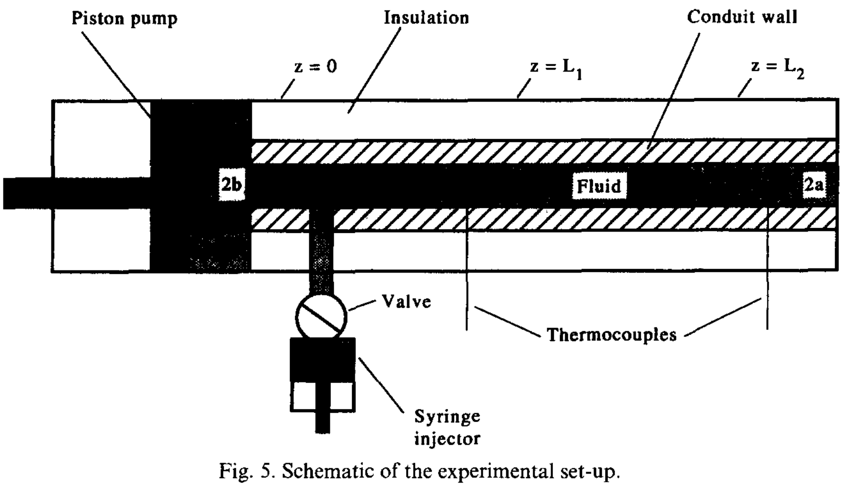


Fig. 5. Schematic of the experimental set-up.

A second set of measurements were obtained when the displacement pump was run at double its original speed so that the Peclet number was now $Pe = 86.6$. Results of this run are summarized in Fig. 7. Again there is excellent agreement with the earlier developed theory, including the lag in arrival time of the thermal pulse relative to the average value of the fluid elements. The transit time of the thermal pulse peak between the two observation points was measured in these runs to be 378 s, yielding a value for $f$ of 0.546. This value for $f$ is very close to that obtained at the lower Peclet number and thus supports the usefulness of this measurement technique for obtaining heat capacity ratios. Also, one can reverse the procedure and use such measurements to determine mean flow velocity in small diameter conduits if the wall and fluid thermal characteristics are known. Such a measurement could be done in a noninvasive manner and thus might find application in the biomedical field where related measuring techniques, termed thermodilution [14], are already used to obtain estimates for cardiac output. It should be pointed out that in thermodilution no account is taken for the heat exchange between the flowing fluid and the vessel walls and this neglect may have to be modified in view of the present results.

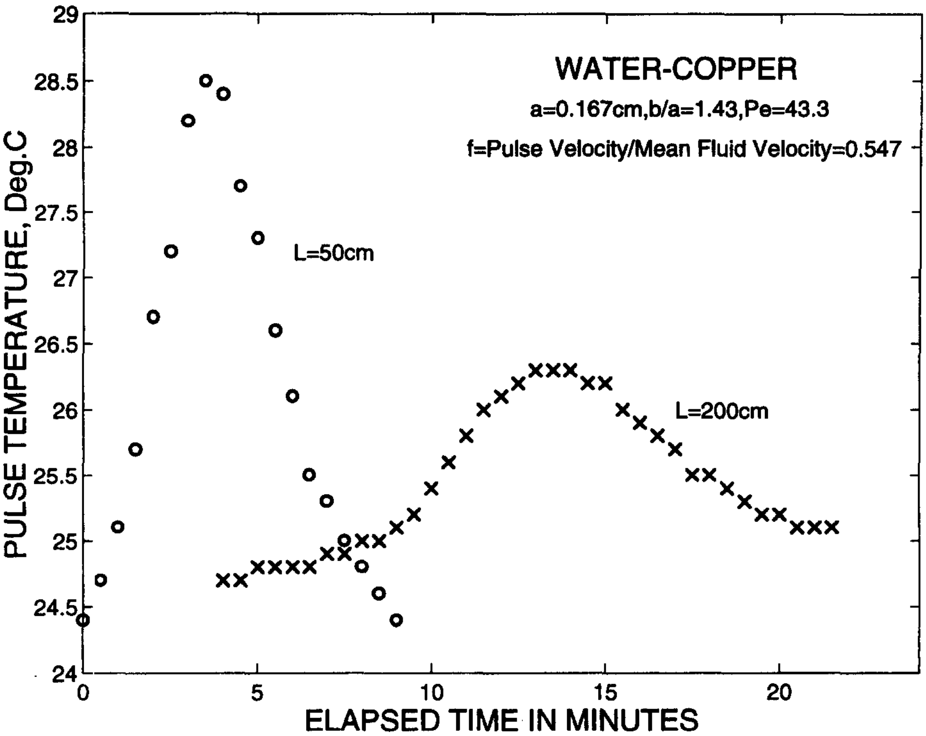


Fig. 6. Measured temporal variation of pulse. Peclet number $Pe$ = 43.3.

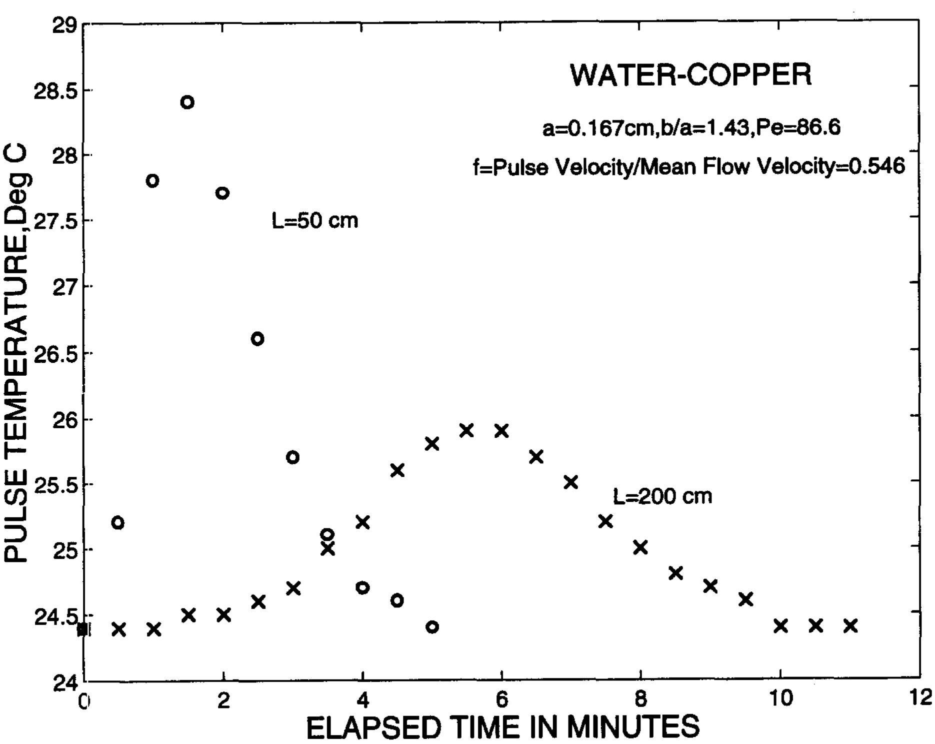


Fig. 7. Experimental measurements of temperature distribution at higher Peclet number of $Pe$ = 86.6.

## 6. CONCLUDING REMARKS

We have examined the propagation and dispersion of thermal pulses in laminar conduit flow, using a multiple time scale expansion, based on a series expansion of the temperature in powers of Graetz number. The analytical results show that the pulses travel down the conduits at sub-mean flow speeds, with the particular velocity dependent on fluid wall heat capacity ratio and wall thickness. The effective thermal diffusivity for these pulses increases with Peclet number and wall thickness and agrees with results for insulating walls, when the wall thickness is allowed to approach zero. Although some small differences are noted in propagation speed and pulse broadening with change from a tube to flat plate geometry, the qualitative behaviour is similar. A set of experiments confirm our theoretical predictions and suggest a way to quickly measure thermal capacity ratios between the conduit wall and the flowing fluid and possibly provide a new approach to noninvasively measure mean flow speeds in small conduits. Also the possibility of enhancing chromatographic separations using heat pulses should not be overlooked.

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